Organizers and speakers
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• randomized algorithms
• uncertain control design
• robust control
• convex optimization
• scenario approach
• sequential methods

Workshop duration
A full day

Abstract
The objective of this workshop is to introduce the attendee to randomized methods for the analysis and design of control systems.
Randomized methods are an emerging technology to address problems that are otherwise difficult to solve along more traditional approaches. The main underlying idea is to replace the normally infinite set of possible uncertainty outcomes with finitely many samples that are representative of the uncertainty domain. One of the main properties which makes randomized methods attractive is that the solutions they provide come accompanied by precise probabilistic guarantees that also refer to unseen cases in the uncertainty domain. This is relevant to probabilistic performance guarantees, as well as to a certified satisfaction of control constraints. Depending on the application at hand, the samples can be drawn from a probabilistic model of uncertainty, or they can be obtained as observations. The latter situation covers data-based approaches in learning and identification.

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Samples can be used all at the same time (batch or scenario approach) or in succession (sequential methods). This workshop covers both situations. The *scenario approach* is a well established paradigm that can be used to solve convex uncertain optimization problems arising in control. *Sequential methods* are instead very well suited for feasibility problems, and the presentation will emphasize the many pros and achievements obtained in this direction. The presentation will be gradual to allow an in-depth understanding of the fundamental concepts. Practical examples will illustrate the main ideas, and a presentation of open problems will complete the workshop.

**Main topics and workshop structure**

A randomized algorithm is an algorithm that makes random choices based on samples extracted from a pre-assigned probability distribution. Hence, the returned numerical results are subject to randomness. The level of randomness, however, can be kept under control by means of many results available in the existing literature.

Randomized algorithms have been successfully used since the forties in diverse scientific disciplines such as computer science, physics, numerical analysis, and optimization. In control, the introduction of randomized methods is more recent, but randomized methods are gaining popularity and spreading rapidly due to their ability to overcome the computational difficulties that arise with more traditional deterministic approaches.

The goal of the workshop is to introduce the attendee to randomized methods for the analysis and design of control systems. The existing methods can be grouped in two categories:

(i) batch (scenario) approach; (ii) sequential methods.

Both will be covered in the workshop.

**Batch approach - the scenario method**

The “scenario approach” [1-3] is a sample-based technique to address uncertain convex optimization problems arising in control design. It offers a viable way to obtain chance-constrained feasible solutions that exhibits empirical optimality. This workshop aims at providing the attendee with a broad introduction to the scenario approach, followed by a more-in-depth presentation of the foundational mathematical aspects at its basis, and some application examples. A detailed description of the main topics is as follows.

The scenario approach to control

Many problems in control are formulated as optimization problems. In many cases, the cost function incorporates variables that are used to model uncertainty, in addition to optimization variables. The scenario approach is first presented as a method to optimize an uncertain cost function. The main idea of sampling from the uncertainty set and optimizing with the sampled scenarios as constraints is introduced.

The scenario approach stands on some fundamental theoretical results [4], which make this approach mathematically solid. The mathematical foundations of the scenario approach will be
reviewed in a self-contained manner.

**Modulating Robustness in Control Design: Principles and Algorithms**

Robust control can lead to designs that are overconservative because all emphasis is placed on safe-guarding against all possible occurrences. When 100% guarantee of robustness is required, standard robust control is indeed the way to go. However, in many applications, robustness in 100% of the cases is not really necessary and it is a fact that accepting a small compromise in robustness guarantees (e.g. accepting a 99% guarantee) can lead to a huge improvement in performance. It will be shown that the scenario approach offers a viable way to find a suitable trade-off between risk and return by constraint removal [5,6].

**Empirical distribution of costs**

In correspondence of the sampled instances of the uncertain parameter, the scenario solution incurs various costs called “empirical costs”, which offer an empirical description of the cost distribution. A precise characterization of the risks associated to the empirical costs, namely an evaluation of the probability that the various empirical costs are exceeded by new situations, is presented [7].

**Sequential Methods**

The sequential randomized algorithms that we study in this workshop provide a generalization of all the sequential methods that have been previously analyzed in the literature for specific convex control problems subject to structured random uncertainty. These problems include the design of linear quadratic regulators and solutions to linear matrix inequalities under uncertainty [8,9].

In particular, we study randomized sequential methods for finding a probabilistic feasible solution (i.e. a solution which holds with high probability) to a convex constraint in the design variables and subject to nonlinear structured random uncertainty [10,11]. The algorithms presented in the literature are based on two fundamental ingredients:

i) an *Oracle*, which should check probabilistic feasibility of a candidate solution;

ii) an *Update Rule*, which exploits the convexity of the problem for constructing a new candidate solution based on the Oracle outcome.

We now describe these two ingredients.

**Probabilistic Oracle**

The Oracle constitutes the randomized part of the algorithm, and its role is to decide probabilistic feasibility of the current solution. This decision is made based on random samples of the uncertainty. One of the objectives is to derive the sample complexity, i.e. the number of random samples to be drawn. We remark that in this case the sample complexity cannot be obtained using classical bounds like the Chernoff bound because the generated samples along the iterations are not statistically independent. We show, however, a simple formula [12,13] which provides the sample complexity, showing that the obtained sample size is independent of the number of uncertain and design parameters, and which depends very mildly (in a logarithmic fashion) on the number of iterations and probabilistic levels. This is one of the key features of these algorithms, which enjoy polynomial-time complexity. For this reason, these algorithms are
said to break the curse of dimensionality, at the expense of a probability of violation.

**Update Rules**

Various update rules have been proposed in the literature. The first one that has been studied is based on a (sub)gradient descent technique [8]. More sophisticated techniques, which improve convergence rates, have been subsequently developed and fall in the class of the so-called random localization methods [14]. These methods require the computation at each step of the algorithm of a center of a suitably constructed localization set, which is then used as the candidate solution of the problem. In particular, in probabilistic cutting plane methods, the localization set is a polytope, and the candidate solution is its analytic center. In the probabilistic ellipsoid algorithm, the localization set is an ellipsoid and the candidate solution is its center.

**Probabilistic Properties**

Several probabilistic properties for these algorithms have been established and are discussed in this workshop [10,11]. In particular, the convergence properties, the required number of iterations, the probability of a successful exit of the algorithm or the derivation of a violation certificate (in case of unsuccessful exit of the algorithm), are studied in great details. We also discuss the differences with other classical approaches developed in the area of stochastic approximation methods.

**A Posteriori Analysis**

Subsequently, we study a posteriori analysis (both probabilistic and deterministic) to establish the goodness of the solution obtained by the proposed sequential randomized algorithm. In particular, for deterministic analysis, we discuss methods based on classical extreme point results developed in robust control, while for probabilistic analysis, Monte Carlo techniques based on random extractions of uncertainty samples are explained.

**Randomized Algorithms Control Toolbox**

Batch and sequential algorithms previously discussed have been implemented in Matlab in the toolbox RACT (Randomized Algorithms Control Toolbox) [15]. This toolbox provides convenient uncertain object manipulation and implementation of these methods. Simulations and numerical examples are performed to show the efficacy of the proposed methodology. The package can be freely downloaded from http://ract.sourceforge.net.

**References**


**Biographies of organizers**

**Marco Campi** is Professor of Automatic Control at the University of Brescia, Italy.

In 1988, he received the Doctor degree in electronic engineering from the Politecnico di Milano, Milan, Italy. From 1988 to 1989, he was a Lecturer at the Department of Electrical Engineering of the Politecnico di Milano. From 1989 to 1992, he was a Research Fellow at the Centro di Teoria dei Sistemi of the National Research Council (CNR) in Milan and, in 1992, he joined the University of Brescia, Brescia, Italy. He has held visiting and teaching appointments at the Australian National University, Canberra, Australia; the University of Illinois at Urbana-Champaign, USA; the Centre for Artificial Intelligence and Robotics, Bangalore, India; the University of Melbourne, Australia; and the Kyoto University, Japan.

Marco Campi is the Chair of the Technical Committee IFAC on Modeling, Identification and Signal Processing (MISP), and was the Chair of the Technical Committee IFAC on Stochastic Systems (SS) from 2002 to 2008. He has been in various capacities on the Editorial Board of Automatica, Systems and Control Letters and the European Journal of Control. Marco Campi is a recipient of the "Giorgio Quazza" prize, and, in 2008, he received the IEEE CSS George S. Axelby outstanding paper award for the article The Scenario Approach to Robust Control Design. He has delivered plenary and semi-plenary addresses at major conferences including SYSID, MTNS, and CDC, and has been a distinguished lecturer of the Control Systems Society. Marco Campi is a Fellow of IEEE, a member of IFAC, and a member of SIDRA.

The research interests of Marco Campi include: system identification, stochastic systems, randomized methods, adaptive and data-based control, robust optimization, and learning theory.

**Fabrizio Dabbene** received the Laurea degree in 1995 and the Ph.D. degree in Systems and Computer Engineering in 1999, both from Politecnico di Torino, Italy. Since 2001 he is with the IEIIT Institute of the National Research Council (CNR) of Italy, where he is currently a Senior Researcher. He has held visiting and research positions at The University of Iowa, at Penn State University and at the RAS (Russian Academy of Sciences) Institute of Control Science, Moscow.

His research interests include probabilistic and randomized methods for systems and control, robust control and identification of complex systems, convex optimization and modeling of environmental systems. On these topics, he has published more than 80 research papers, which include 30 articles published in international journals. He is co-author of the book *Randomized Algorithms for Analysis and Control of Uncertain Systems*, Springer-Verlag, published in two editions, and editor of the book *Probabilistic and Randomized Methods for Design under Uncertainty*, also published by Springer. He has been co-organizer of several invited sessions and special courses, and of three Conference Workshops at CDC-00, MSC-08 and MSC-10. He is a recipient of the *Outstanding Paper Award from EurAgeng* in 2010. He serves as Associate Editor for Automatica and as Chair of the IFAC Technical Committee on Robust Control.

Dr. Dabbene is a Senior Member of the IEEE, and he has taken various responsibilities within the IEEE Control Systems Society. He has been an Associate Editor for the IEEE Transactions on Automatic Control (2008-2012), Program Chair for the CACSD Symposium of the 2010 IEEE Multiconference on Systems and Control, Chair of the IEEE Technical Committee on
CACSD (2010-present), member of CEB (2002-2008), IPC member of various IEEE conferences, and member of several CDC Best Student Paper Award Committees. He is elected member of the IEEE-CSS Board of Governors for the years 2014-2016.

**Simone Garatti** is currently an Assistant Professor in the Automatic Control area (SSD INF-INF/04) at the Dipartimento di Elettronica, Informazione e Bioingegneria of the Politecnico di Milano. He was born in Brescia, Italy, in 1976. In 2000, he received the Laurea degree cum laude in Computer Science Engineering from the Politecnico di Milano and in 2004 he received the Ph.D. cum laude in Information Technology Engineering from the Politecnico di Milano. In 2004, he was a research fellow at the Dipartimento di Elettronica ed Informazione of the Politecnico di Milano. He has been assistant professor within the same department since 2005.

During his career, Simone Garatti was visiting scholar at the Lund University of Technology, Lund, Sweden in 2003 and at the University of California San Diego (UCSD), San Diego, CA, USA in 2006. Moreover, in 2007, he was invited for a short-term visit at the Massachusetts Institute of Technology and the Northeastern University, Boston, MA, USA, during which he gave some invited lectures on his research topics. Simone Garatti is member of the IEEE Control System Society Technical Committee on Computational Aspects of Control System Design and of the IFAC Technical Committee on Modelling, Identification and Signal Processing.

The Simone Garatti’s research interests include system identification and model quality assessment, identification of interval predictor models, randomized optimization for problems in systems and control. He is author of about 40 contributions on international journals, international books, and proceedings of international conferences.

**Maria Prandini** is Associate Professor in Control at the Dipartimento di Elettronica, Informazione e Bioingegneria (DEIB) of the Politecnico di Milano. Italy.

She obtained the Laurea degree in Electrical Engineering (Politecnico di Milano 1994), and the Ph.D. degree in Information Technology (Università degli Studi di Brescia 1998) with a dissertation on adaptive control of stochastic systems. After receiving her Ph.D., she was a visiting postdoctoral researcher at the Department of Electrical Engineering and Computer Sciences, University of California at Berkeley, from 1998 to 2000. She also held visiting positions at Delft University of Technology (1998), Cambridge University (2000), UC Berkeley (2005), and ETH Zurich (2006). From December 2002 to January 2011, she was Assistant Professor at DEIB, Politecnico di Milano.

She is currently serving as associate editor of the IEEE Transactions on Automatic Control, the IEEE Transactions on Control Systems Technology, and Nonlinear Analysis: Hybrid Systems. She is also member of the IFAC Technical Committee on Discrete Event and Hybrid Systems and member of the IEEE Control Systems Society Conference Editorial Board. She was discussion editor of the European Journal of Control from 2007 to 2013, and member of the IFAC Technical Committee on Stochastic Systems from 2003 to 2008. She has been IPC member of various conferences. From January 2013, she is editor of the electronic publications of the IEEE Control Systems Society.

Her research interests include: stochastic and hybrid systems, randomized methods, constrained control, model reduction, power networks, and air traffic management. She published about 90 peer-reviewed papers on these topics, and acted as principal investigator in two EC-funded projects entitled “Safety, Complexity and Responsibility based
design and validation of highly automated Air Traffic Management” (FP6, 2007–2011), and “Modelling, verification and control of complex systems: From foundations to power network applications” (FP7, 2010–2013).

**Roberto Tempo** is currently a Director of Research of Systems and Computer Engineering at the institute IEIIT, National Research Council (CNR), Politecnico di Torino, Italy. He has held visiting and research positions at various institutions, including Chinese Academy of Sciences in Beijing, Kyoto University, The University of Tokyo, University of Illinois at Urbana-Champaign, German Aerospace Research Organization in Oberpfaffenhofen and Columbia University in New York.

Dr. Tempo’s research activities are mainly focused on the analysis and design of complex systems with uncertainty, and various related applications within information technology, which currently include the PageRank computation in the Google search engine, distributed localization of wireless sensor networks and asynchronous opinion dynamics in social networks. On these topics, he has published more than 180 research papers in international journals, books and conferences. He is also a co-author of the book *Randomized Algorithms for Analysis and Control of Uncertain Systems*, Springer-Verlag, London, published in two editions in 2005 and 2013.

He is a Fellow of the IEEE for “Contributions to Robust Identification and Control of Uncertain Systems” and a Fellow of the International Federation of Automatic Control (IFAC) for “Contributions to the Analysis and Control of Uncertain Systems, for Pioneering the Probabilistic Approach to Robustness.” He is a recipient of the *Outstanding Paper Prize Award* from the IFAC for a paper published in the journal Automatica, and of the *Distinguished Member Award* from the IEEE Control Systems Society. He is a Corresponding Member of the Academy of Sciences, Institute of Bologna, Italy, Class Physical Sciences, Section Technical Sciences.

In 2010 Dr. Tempo has served the IEEE Control Systems Society as President. He is General Co-Chair for the 52nd IEEE Conference on Decision and Control, to be held in Florence, Italy, in 2013 and he was Program Chair of the first joint IEEE Conference on Decision and Control and European Control Conference, Seville, Spain, 2005. He is currently an Editor and Deputy Editor-in-Chief of Automatica, a Senior Editor of the IEEE Transactions on Automatic Control, a member of the Advisory Committee of the IEEE Transactions on Control of Network Systems, and an Editor at Large of the Asian Journal of Control. He has been Editor for Technical Notes and Correspondence of the IEEE Transactions on Automatic Control in 2005–2009.